Ingleton Primary School

Calculation Policy for Mathematics

December 2018

Ingleton Primary School - Calculations Policy

This policy is based on the National Curriculum 2014 and helps to develop the three main aims of **Fluency**, **Reasoning** and **Problem Solving.** It is designed to give pupils a consistent and smooth progression of learning when using the four main operations.

The calculations policy is organised according to age expectations as set out in the New National Curriculum 2014, however, it is vital that pupils are taught according to the stage they are currently working at.

Please note that early teaching and learning in Reception follows the 'EYFS' document.

Aims

⇒ To form a core set of methods that every child will use and build upon.

⇒ To ensure greater consistency in the teaching of written calculations & fraction calculations within school.

⇒ To strengthen continuity and progression of children's understanding of calculations.

⇒ To build upon and ensure continued use of models and images, visual, concrete and abstract, to develop children's conceptual understanding.

⇒ To be a guide on the teaching of calculation skills for teachers, teaching assistants and parents.

⇒ Mental and written calculation methods should be taught alongside each other throughout the entirety of this progression. When teaching children to calculate emphasis should be placed on choosing and using the method that is most efficient.

⇒ If a child can complete a calculation mentally or with jottings, they should not be expected to complete a written algorithm.

⇒ Whilst no longer part of the statutory curriculum, children should also be taught when and how to use a calculator appropriately.

Important points to note:

⇒ This booklet is a progression in calculations and any of these methods can and should be utilised in other year groups to support or extend pupils as their personal learning requires

⇒ It is school policy that when 'carrying', the carried numbers should be shown under the bottom equals line (with the exception of long multiplication in which carried numbers will be shown above the calculation).

⇒ Pupils should use the 'one digit, one square' mantra when using written methods and when calculating with decimals, the decimal point **should not** have its own square as it has no inherent place value; it should instead straddle the line between the units/ ones column and the tenths column.

Addition +

In developing a written method for addition, it is important that children understand the concept of addition, in that it is:

Combining two or more groups to give a total or sum
Increasing an amount

They also need to understand and work with certain principles, i.e. that it is:

the inverse of subtraction
commutative i.e. 5 + 3 = 3 + 5
associative i.e. 5 + 3 + 7 = 5 + (3 + 7)

The fact that it is commutative and associative means that calculations can be rearranged, e.g. 4 + 13 = 17 is the same as 13 + 4 = 17.

Year R

Early Learning Goal:

Using quantities and objects, children add two single-digit numbers and count on to find the answer.

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of practical equipment, including small world play, role play, counters, cubes etc.

Counting all method

Children will begin to develop their ability to add by using practical equipment to count out the correct amount for each number in the calculation and then combine them to find the total. For example, when calculating 4 + 2, they are encouraged to count out four counters and count out two counters.

To find how many altogether, touch and drag them into a line one at a time whilst counting.

By touch counting and dragging in this way, it allows children to keep track of what they have already counted to ensure they don't count the same item twice.

Year R

Addition +

Early Learning Goal:

Using quantities and objects, children add two single-digit numbers and count on to find the answer.

Counting on method

To support children in moving from a counting all strategy to one involving counting on, children should still have two groups of objects but one should be covered so that it cannot be counted. For example, when calculating 4 + 2, count out the two groups of counters as before.

then cover up the larger group with a cloth.

For most children, it is beneficial to place the digit card on top of the cloth to remind the children of the number of counters underneath. They can then start their count at 4, and touch count 5 and 6 in the same way as before, rather than having to count all of the counters separately as before.

Those who are ready may record their own calculations.

Year 1

Add 1 digit and 2 digit numbers to 20 including 0

Addition +

+ = signs and missing numbers

Children need to understand the concept of equality when using the '=' sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'.

2 = 1+ 1

2 + 3 = 4 + 1

Missing numbers need to be placed in all possible places.

3 + 4 = • • = 3 + 4

3 + • = 7 7 = • + 4

Add with numbers up to 20

Use numbered number lines to add, by counting on in ones. Encourage children to start with **larger** number and count on..

The labelled number line

- Children begin to use numbered lines to support their calculations counting on in ones.

To help children develop a sound understanding of numbers and to be able to use them confidently in calculation, there needs to progression in their use of number tracks and number lines .

Year 1

Add 1 digit and 2 digit numbers to 20 including 0

Addition +

In addition to their work with numberlines, children should also become familiar with the use of 'Base 10' practical equipment as this will be utilised later in the progression.

Children will continue to use practical equipment, combining groups of objects to find the total by counting all or counting on. Using their developing understanding of place value, they will move on to be able to use Base 10 equipment to make teens numbers using separate tens and units.

For example, when adding 11 and 5, they can make the 11 using a ten rod and a unit.

The units can then be combined to aid with seeing the final total, e.g.

so 11 + 5 = 16. If possible, they should use two different colours of base 10 equipment so that the initial amounts can still be seen.

Children should:

Have access to a wide range of counting equipment, everyday objects, number tracks and number lines, and be shown numbers in different contexts.

Read and write the addition (+) and equals (=) signs within number sentences.

Interpret addition number sentences and solve missing box problems, using concrete objects and number line addition to solve them: 8 + 3 = ☐

15 + 4 = ☐ 5 + 3 + 1 = ☐ ☐ + ☐ = 6

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most,

count on, number line

Key skills for addition at Y1:

Read and write numbers to 100 in numerals, incl. 1—20 in words

Recall bonds to 10 and 20, and addition facts within 20

Count to and across 100

Count in multiples of 1 2, 5 and 10

Solve simple 1-step problems involving addition, using objects, number lines and pictorial representations.

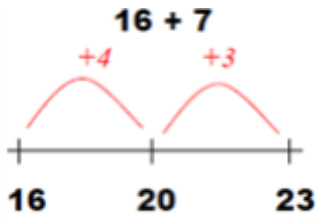
Year 2

Addition +

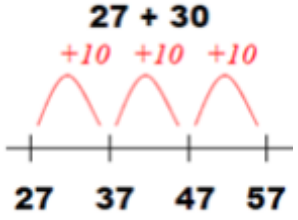
Add with 2-digit numbers

Developing mental fluency with addition and place value involving 2-digit numbers, then establish more formal methods such as those exemplified in the Y3 section

Add 2-digit numbers and units



Add 2-digit numbers and tens.



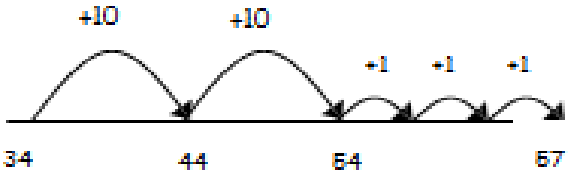
The empty number line is a representation of a mental strategy

Use empty number lines, concrete equipment, hundred squares etc. to build confidence and fluency in mental addition skills.

Add pairs of 2-digit numbers.

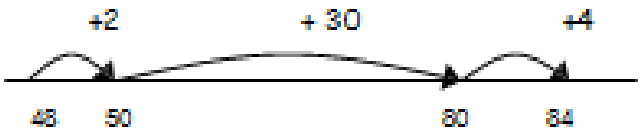
First counting on in tens and ones.

34 + 23 = 57



48+36

or



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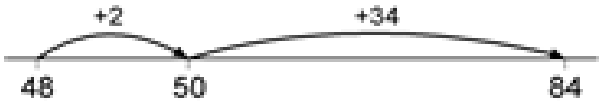


Year 2

Addition +

Add with 2-digit numbers

Developing mental fluency with addition and place value involving 2-digit numbers, then establish more formal methods such as those exemplified in the Y3 section

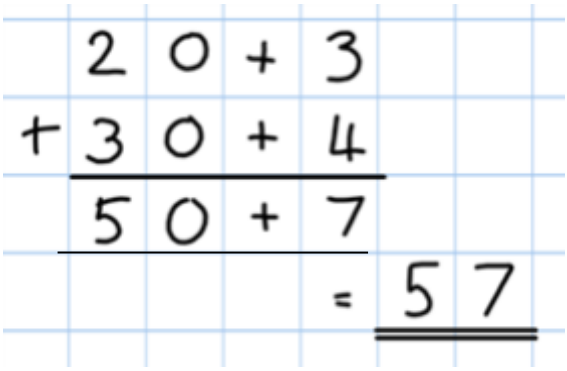
Solve addition problems using the column method involving 2 digit numbers.

Partitioning into tens and ones to lead to a formal written method

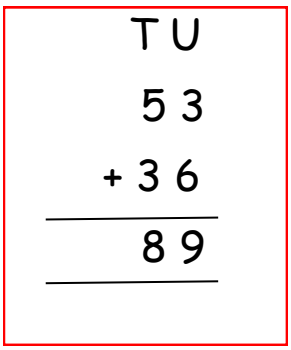
- The next stage is to record mental methods using partitioning into tens and ones separately

Only provide examples that do **NOT** cross the tens boundary until they are secure with the method itself.

Step 1



Step 2



Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary

Key skills for addition at Y2:

Add a 2-digit number and ones (e.g. 27 + 6)

Add a 2-digit number and tens (e.g. 23 + 40)

Add pairs of 2-digit numbers (e.g. 35 + 47) Add three single-digit numbers (e.g. 5 + 9 + 7)

Show that adding can be done in any order (the commutative law).

Recall bonds to 20 and bonds of tens to 100 (30 + 70 etc.)

Count in steps of 2, 3 and 5 and count in tens from any number.

Understand the place value of 2-digit numbers (tens and ones)

Compare and order numbers to 100 using < > and = signs. Read and write numbers to at least 100 in numerals and words.

Solve problems with addition, using concrete objects, pictorial representations, involving numbers, quantities and measures, and applying mental and written methods.

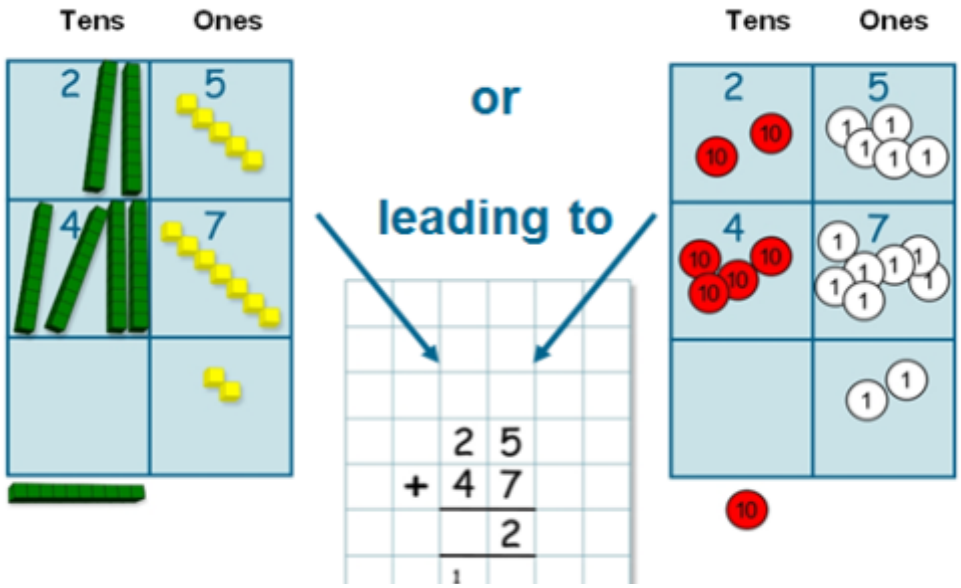
Year 3

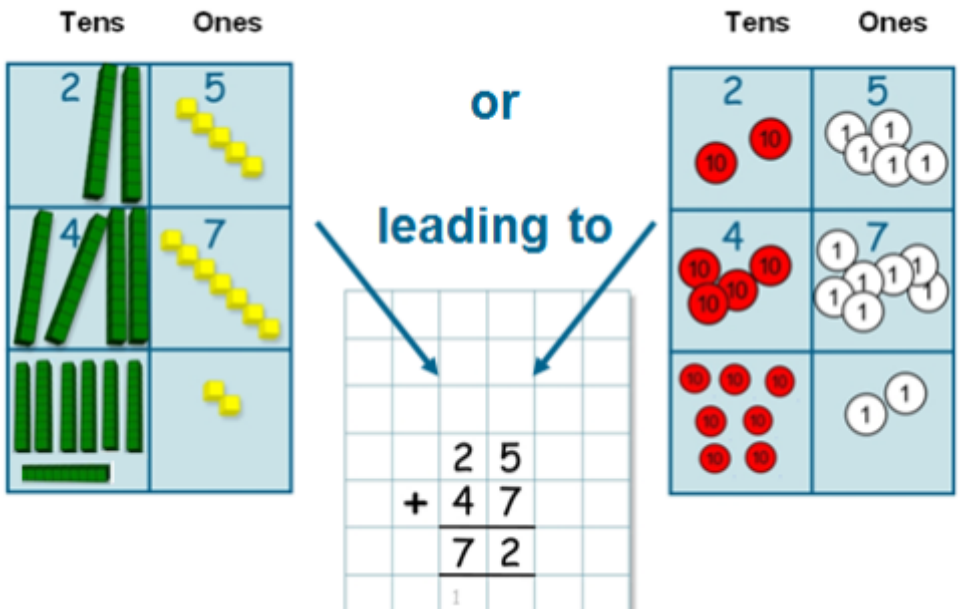
Add numbers with up to 3-digits

Addition +

To ensure the statutory final written method is grounded in understanding, this stage connects the practical equipment to the formal written method.

Children first experience the practical version of column addition and when confident in explaining this, including exchanging when crossing the tens barrier with ones, they record the written method alongside.





Represented in place value columns and rows. Starting adding with the 'least significant digit' . When the tens barriers is crossed in the 'ones' exchange then takes place.

Addition +

Children must have a strong understanding of place value to complete this stage.

Tips:

- Add **units/ones** first.
- Remember the actual value is "two tens add four tens", not "two add four", which equals six tens.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on,

number line, sum, tens, units, partition, plus, addition, column, tens boundary, **hundreds boundary, increase, vertical,**

exchange,

Key skills for addition at Y3:

Read and write numbers to 1000 in numerals and words.

Add 2-digit numbers mentally, incl. those exceeding 100.

Add a three-digit number and ones mentally (175 + 8)

Add a three-digit number and tens mentally (249 + 50)

Add a three-digit number and hundreds mentally (381 + 400)

Estimate answers to calculations, using inverse to check answers.

Solve problems, including missing number problems, using number facts, place value, and more complex addition.

Recognise place value of each digit in 3-digit numbers (hundreds, tens, ones.)

Continue to practise a wide range of mental addition strategies, ie. number bonds, adding the nearest multiple of 10, 100, 100 and adjusting, using near doubles, partitioning and recombining.

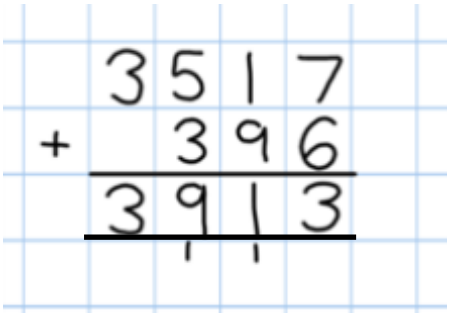
Year 4

Add numbers with up to 4-digits

Addition +

Also include money and measure contexts.

In this method, recording is reduced further. Carried digits are recorded, using the words 'carry ten' or 'carry one hundred' etc., according to the value of the digit.



Reinforce correct place value by reminding them the actual value is 5 hundreds add 3 hundreds, **not 5 add 3**, for example.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on,

number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary,

increase, vertical, exchange, thousands, hundreds, digits, inverse

Addition +

Key skills for addition at Y4:

Select most appropriate method: mental, jottings or written and explain why.

Recognise the place value of each digit in a four-digit number.

Round any number to the nearest 10, 100 or 1000.

Estimate and use inverse operations to check answers.

Solve 2-step problems in context, deciding which operations and methods to use and why.

Find 1000 more or less than a given number.

Continue to practise a wide range of mental addition strategies, ie. number bonds, add the nearest multiple of 10, 100, 1000 and adjust, use near doubles, partitioning and recombining.

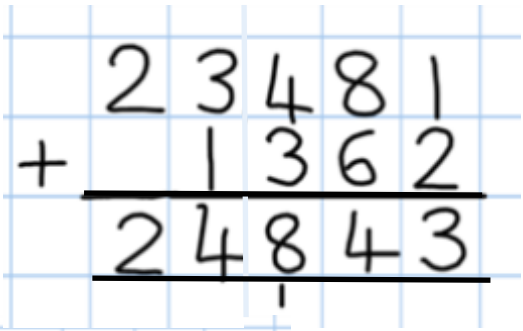
Add numbers with up to 4 digits using the formal written method of column addition

Solve 2-step problems in contexts, deciding which operations and methods to use and why.

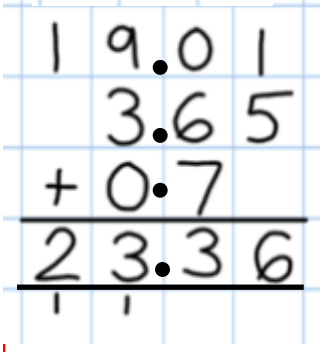
Estimate and use inverse operations to check answers to a calculation.

Addition +

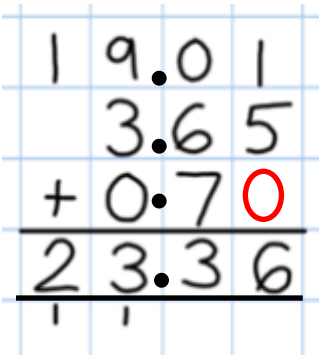
Year 5 Add numbers with more than 4 digits, more than 2 numbers.



Numbers should exceed 4 digits.



Pupils should be able to add more than two values, carefully aligning place value columns.



Empty places should be filled with zero to show the place value.

Say "6 tenths add 7 tenths" value in each column.

Children should understand the place value of tenths and hundredths and use this to align numbers with different numbers of decimal places.

Decimals used in the context of money and measures.

Addition +

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, exchange, vertical, thousands, hundreds, digits, inverse & **decimal places, decimal point, tenths, hundredths, thousandths**

Key skills for addition at Y5:

Add numbers mentally with increasingly large numbers, using and practising a range of mental strategies i.e. add the nearest multiple of 10, 100, 100 and adjust; use near doubles, inverse, partitioning and recombining; using number bonds.

Use rounding to check answers and accuracy.

Solve multi-step problems in contexts, deciding which operations and methods to use and why.

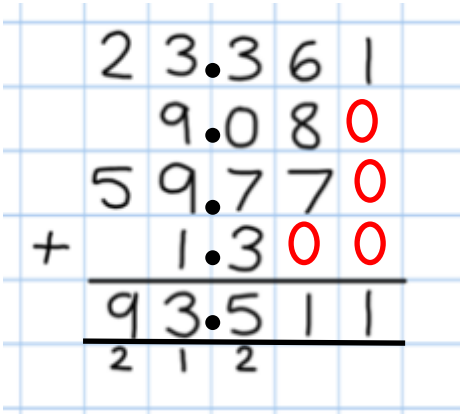
Read, write, order and compare numbers to at least 1 million and determine the value of each digit.

Round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000.

Add numbers with more than 4 digits using formal written method of column addition.

Year 6 Add several numbers of increasing complexity

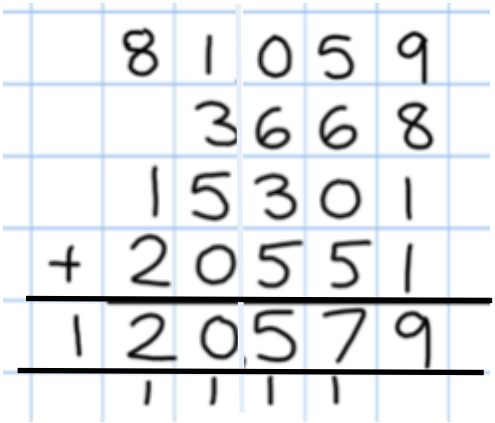
Addition +



Adding several numbers with different numbers of

decimal places (including money and measures): Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row.

Zeros should be added into any empty decimal places, to show there is no value to add.



Adding several numbers with more than 4 digits

Addition +

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, exchange, vertical, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths

Key skills for addition at Y6:

Perform mental calculations, including with mixed operations and large numbers, using and practising a range of mental strategies.

Solve multi-step problems in context, deciding which operations and methods to use and why.

Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.

Read, write, order and compare numbers up to 10 million and determine the value of each digit.

Round any whole number to a required degree of accuracy.

Pupils understand how to add mentally with larger numbers and calculations of increasing

complexity.

Subtraction -

Subtraction

In developing a written method for subtraction, it is important that children understand the concept of subtraction, in that it is:

Removal of an amount from a larger group (take away)
Comparison of two amounts (difference)

They also need to understand and work with certain principles, i.e. that it is:

the inverse of addition
not commutative i.e. 5 - 3 is not the same as 3 - 5
not associative i.e. 10 - 3 - 2 is not the same as 10 - (3 – 2)

Subtraction -

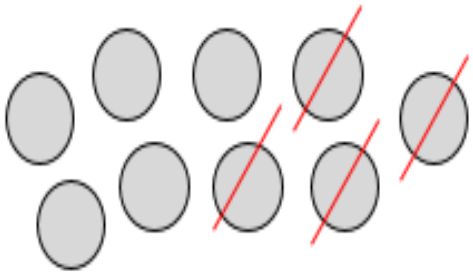
Year R Early Learning Goal:

Using quantities and objects, children subtract two single-digit numbers and count on or back to find the answer.

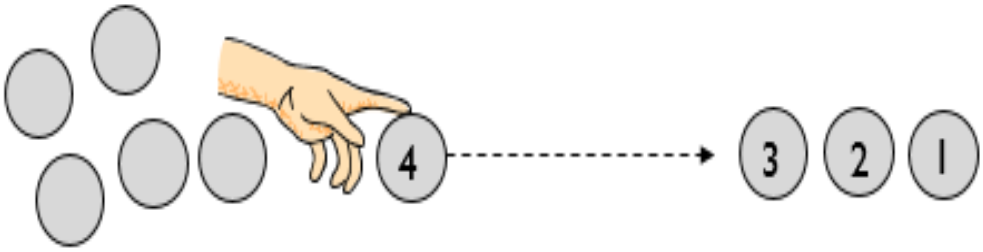
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of practical equipment, including small world play, role play, counters, cubes etc

Taking away

Children will begin to develop their ability to subtract by using practical equipment to count out the first number and then remove or take away the second number to find the solution by counting how many are left e.g. 9 – 4.



For illustration purposes, the amount being taken away are show crossed out. Children would be encouraged to physically remove these using touch counting.



By touch counting and dragging in this way, it allows children to keep track of how many they are removing so they don't have to keep recounting. They will then touch count the amount that are left to find the answer.

Those who are ready may record their own calculations.

Year 1 **Subtract from numbers up to 20**

Subtraction -

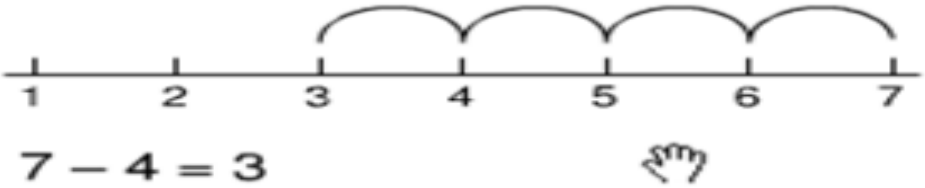
Begins with the use of concrete objects and pictorial representations. If appropriate, progress from using number lines with every number shown to number lines with significant numbers shown.

Subtract by taking away

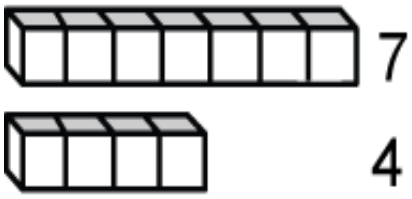
Count back in ones on a numbered number line to take away, with numbers up to 20:

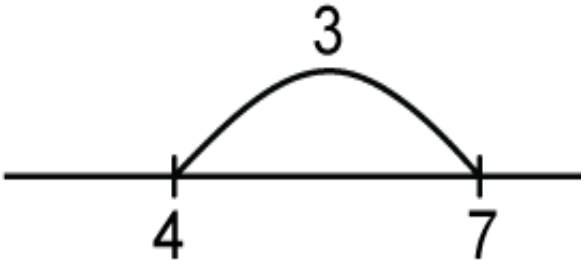
Model subtraction using hundred squares and numbered number lines.

lines/tracks and practically.



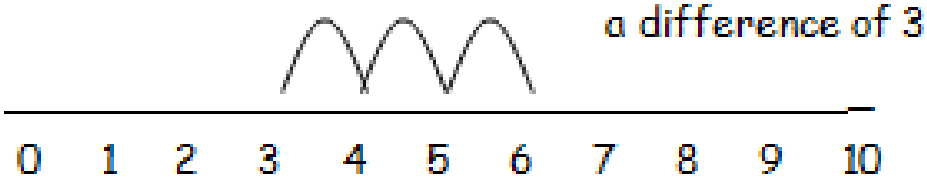
Subtract by finding the difference between





The difference between 7 and 4 is 3.

6-3 =3



Year 1 **Subtract from numbers up to 20**

Subtraction -

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how

many more, how many fewer / less than, most, least, count back , how many left, how much less is_?

Key skills for subtraction at Y1:

☐ Given a number, say **one more or one less.**

☐ Count to and over 100, **forward and back**, from any number.

☐ Represent and use **subtraction facts to 20 and within 20**.

☐ Subtract with **one-digit and two-digit** numbers to 20, including zero.

☐ Solve one-step problems that involve addition and subtraction, using concrete objects (ie bead string,

objects, cubes) and pictures, and missing number problems.

☐ Read and write numbers from 0 to 20 in numerals and words.

Subtraction -

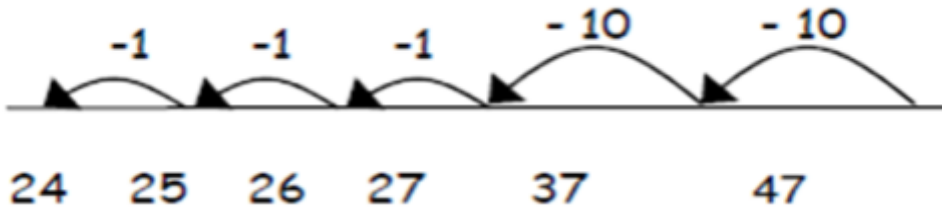
Year 2 Subtract with 2-digit numbers

Recognise subtraction cannot be done in any order.

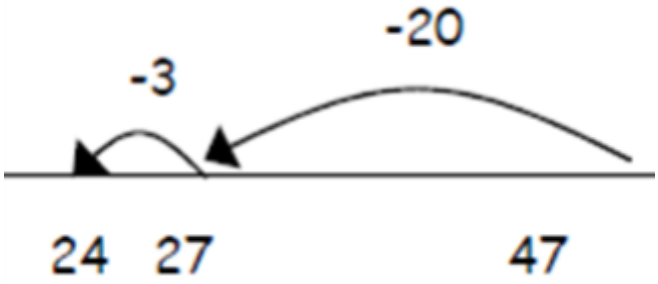
Subtract on a number line by counting back, aiming to develop mental subtraction skills.

Subtracting pairs of 2-digit numbers on a number line:

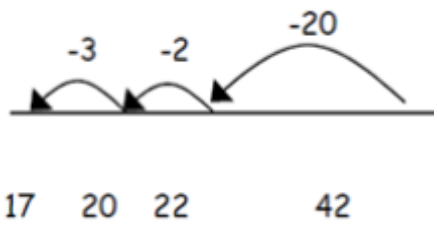
47 - 23 = 24 Partition the second number and subtract it in tens and units, as below:



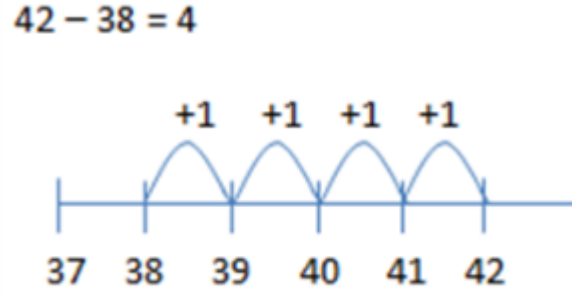
Move towards more efficient jumps back, as below:



Teaching children to **bridge through ten** can help them to become more efficient, for example 42—25:



Mental strategy - subtract numbers close together by **counting on:**

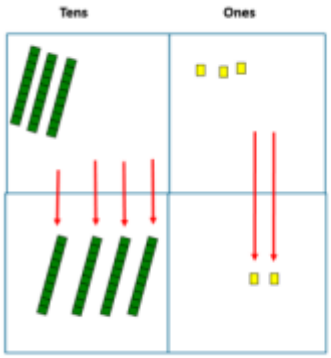


Many mental strategies are taught. Children are taught to recognise that when numbers are close together, it is more efficient to **count on** the difference. They need to be clear about the relationship between addition and subtraction.

Subtraction -

Year 2 Subtract with 2-digit numbers

Introduce **partitioned column subtraction** method.



	70	5
-	40	2
	30	3

With NO exchange

The concept of exchange with this method can be introduced as per the Y3 progression if children are ready and in need of extension.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_?

difference, count on, strategy, partition, tens, units

Key skills for subtraction at Y2:

☐ Recognise the place value of each digit in a two-digit number.

☐ Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100.

☐ Subtract using concrete objects, pictorial representations, 100 squares and mentally, including: a two digit number and ones, a two-digit number and tens, and two two-digit numbers.

☐ Show that subtraction of one number from another cannot be done in any order.

☐ Recognise and use inverse relationship between addition and subtraction, using this to check calculations

and missing number problems.

☐ Solve simple addition and subtraction problems including measures, using concrete objects, pictorial representation, and also applying their increasing knowledge of mental and written methods.

☐ Read and write numbers to at least 100 in numerals and in words.

Year 3 Subtracting with 2-digit and 3 digit numbers

Subtraction -

Practical equipment using exchange to 'take away'

Children use practical apparatus to take away the smaller number from the larger. This should be used to model exchanging as in the example.

<u>Progress through these manipulatives should be guided by understanding not age or year group. The year group simply refers to the expectations in number.</u>

Children use practical apparatus to take away the smaller number from the larger.
Children's place value knowledge should be good enough to understand that the change still represents the original starting number and is just a different way of partitioning it

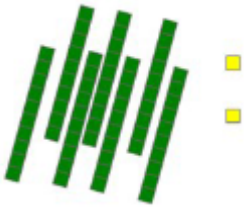
Children first experience the practical version of column subtraction and when confident in explaining this, including exchanging when 'not having enough to subtract from', they record the written method alongside.

70 + 2

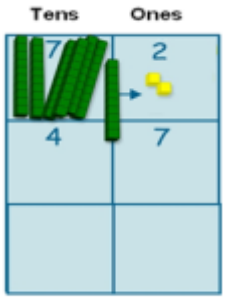
\- 40 + 7

20 + 5 = **25**

72 - 47



Before subtracting '7'units from the 72 blocks, they will need to exchange a row of 10 for ten units. Then subtract the 7 units and subtract 4 tens.



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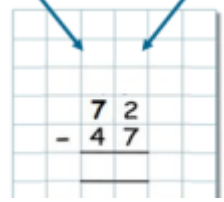
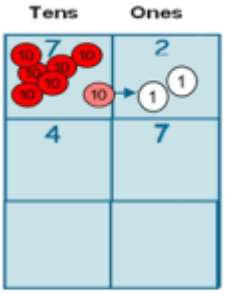
leading to





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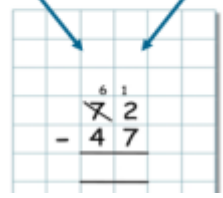
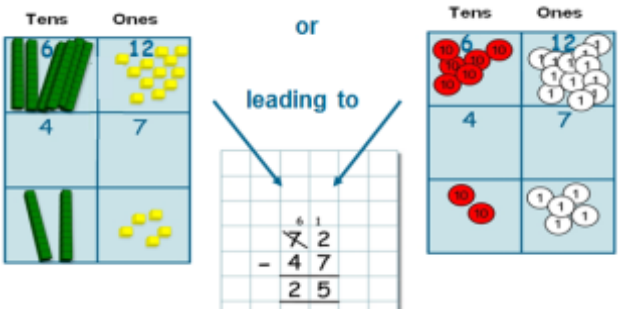


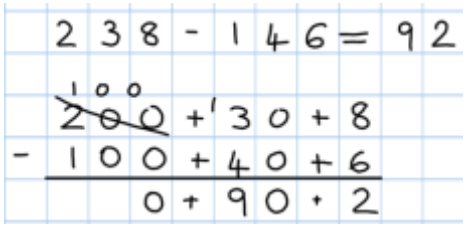
Year 3 Subtracting with 2-digit and 3 digit numbers

Subtraction -

Following on from the previous page, this is how pupils would work through the method.



This is for the purpose pf practical learning and the images here show the total existing alongside the original number, it is suggested that the 47 would be physically 'removed' from the original set (72), before then 'dragging' what is left down to the totals box.



When ready to move on from the practical methods, pupils can move onto an expanded written algorithm .

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more,

how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count

on, strategy, partition, tens, units **exchange, decrease, hundreds, value, digit**

Key skills for subtraction at Y3:

☐ Subtract mentally a: **3-digit number and ones, 3-digit number and tens, 3-digit number and hundreds** .

☐ Estimate answers and use inverse operations to check.

☐ Solve problems, including missing number problems.

☐ Find 10 or 100 more or less than a given number.

☐ Recognise the place value of each digit in a 3-digit number .

☐ Counting up differences as a mental strategy when numbers are close together or near multiples of 10 (see examples above)

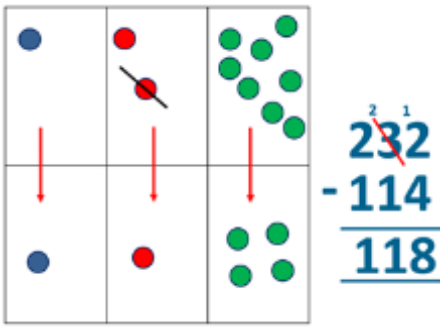
☐ Read and write numbers up to 1000 in numerals and words.

☐ Practise mental subtraction strategies, such as subtracting near multiples of 10 and adjusting (e.g. subtracting 19 or

21), and select most appropriate methods to subtract, explaining why.

Year 4 Subtract with up to 4-digit numbers

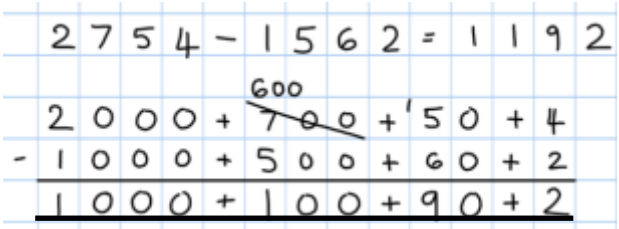
Subtraction -

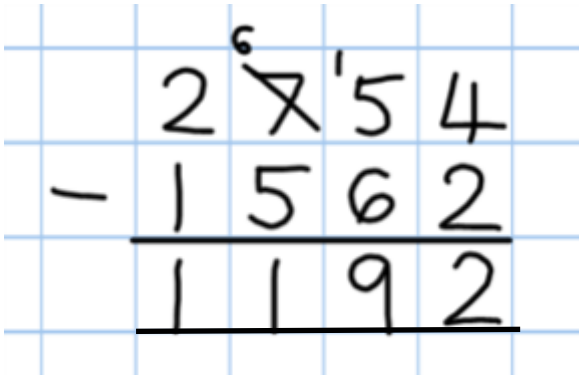


Moving towards more complex numbers and values. Use **place value counters** to reinforce "exchanging" when needed.

Ensure you give plenty of opportunities to apply this to money and measures.

Start with this method, supported by Base 10 for those who need reinforcement with exchanging





Moving onto this more efficient method once pupils are ready—this is the method they will be expected to use throughout the rest of the Key Stage so the more practice they have, the better!

Always encourage children to consider the best method for the numbers involved— mental, counting on, counting back or written method.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between,

how many more, how many fewer / less than, most, least, count back , how many left, how much less is? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit,

Year 4 Subtract with up to 4-digit numbers

Subtraction -

Key skills for subtraction at Y4:

☐ Subtract by counting on where numbers are close together or they are near to multiples of 10, 100 etc.

☐ Children select the most appropriate and efficient methods for given subtraction calculations.

☐ Estimate and use inverse operations to check answers.

☐ Solve addition and subtraction 2-step problems, choosing which operations and methods to use and why.

☐ Solve simple measure and money problems involving fractions and decimals to two decimal places.

☐ Find 1000 more or less than a given number.

☐ Count backwards through zero, including negative numbers.

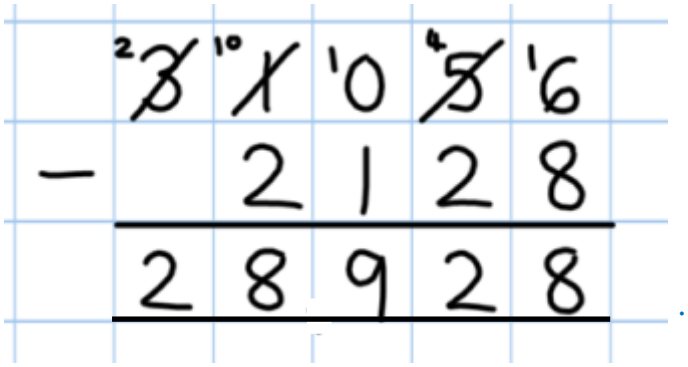
☐ Recognise place value of each digit in a 4-digit number Round any number to the nearest 10, 100 or 1000

☐ Solve number and practical problems that involve the above, with increasingly large positive numbers.

Subtraction -

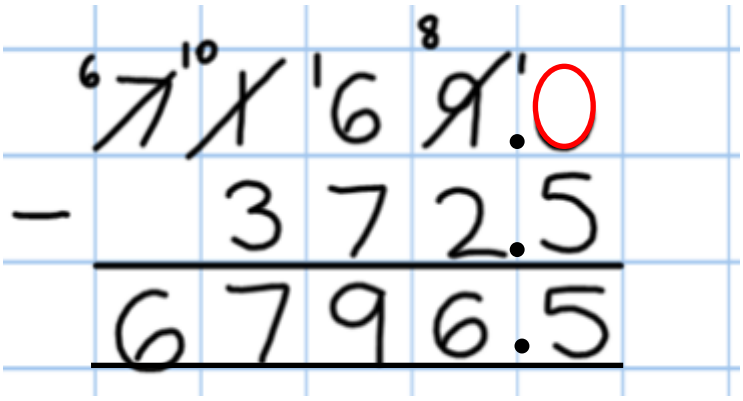
Year 5 Subtract with at least 4 digit numbers.

Subtracting with larger integers, including money, measures, decimals.



Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point.

For a calculation like 7169 - 372.5 where one number in the calculation has a dp but the other doesn't, children should use a place holder as illustrated here in the red (*pupils need not use red, it is for illustrative purposes only)*



Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance

between, how many more, how many fewer / less than, most, least, count back , how

many left, how much less is_? difference, count on, strategy, partition, tens, units

exchange, decrease, hundreds, value, digit, inverse, **tenths, hundredths, decimal point, decimal**

Year 5 Subtract with at least 4 digit numbers

Subtraction -

Key skills for subtraction at Y5:

☐ Subtract numbers mentally with increasingly large numbers .

☐ Use rounding and estimation to check answers to calculations and determine, in a range of contexts,

levels of accuracy .

☐ Solve addition and subtraction multi-step problems in context, deciding which operations and methods

to use and why.

☐ Read, write, order and compare numbers to at least 1 million and determine the value of each digit.

☐ Count forwards or backwards in steps of powers of 10 for any given number up to 1 million.

☐ Interpret negative numbers in context, counting forwards and backwards with positive and negative integers

through 0.

☐ Round any number up to 1 million to the nearest 10, 100, 1000, 10 000 and 100 000.

Subtraction -

Year 6 Subtracting with increasingly large and more complex numbers and decimal values.

	$^{0}\cancel{1}$	$^{14}\cancel{5}$	$^{9}\cancel{1}0$	$^{1}6$	9	9
−		8	9	9	4	9
		6	0	7	5	0

Using the compact column method to subtract more complex integers.

Using the compact column method to subtract money and measures, including decimals with different numbers of decimal places.

	$^{0}\cancel{1}$	$^{9}\cancel{1}0$	$^{1}5$.	$^{3}\cancel{4}$	$^{1}1$	9	kg
−		3	6 .	0	8	0	kg
		6	9 .	3	3	9	kg

Pupils should be able to apply their knowledge of a range of mental strategies, mental recall skills, and informal and formal written methods when selecting the most appropriate method to work out subtraction problems.

When subtracting decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20

Multiplication x

Multiplication

In developing a written method for multiplication, it is important that children understand the concept of multiplication, in that it is:

repeated addition

They should also be familiar with the fact that it can be represented as an array

They also need to understand and work with certain principles, i.e. that it is:

the inverse of division
commutative i.e. 5 x 3 is the same as 3 x 5
associative i.e. 2 x 3 x 5 is the same as 2 x (3 x 5)

Year R **Early Learning Goal:**

Children solve problems, including doubling.

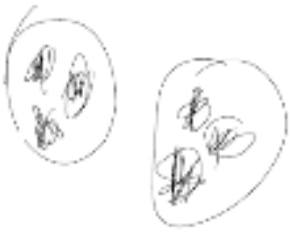
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, including small world play, role play, counters, cubes etc.

Children may also investigate putting items into resources such as egg boxes, ice cube trays and baking tins which are arrays.

They may develop ways of recording calculations using pictures, etc.

A child's jotting showing the fingers on each hand as a double.

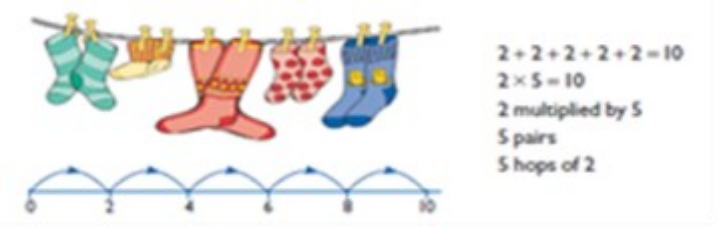
A child's jotting showing double three as three cookies on each plate.



Year 1

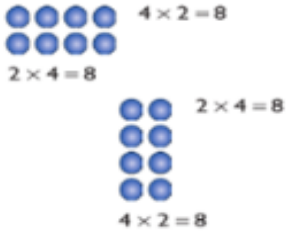
Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Multiplication ×



5+5+5+5+5+5 = 30

6x5 =30



Start to explore arrays to show multiplication can be done in any order. These arrays could be introduced through the use of everyday items such as egg boxes or through mathematical equipment such as Numicon.

Give children experience of counting equal group of objects in 2s, 5s and 10s.

☐ Present practical problem solving activities involving

counting equal sets or groups, as above.

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count

Key skills for multiplication at Y1:

Count in multiples of 2, 5 and 10.

Solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Make connections between arrays, number patterns, and counting in twos, fives and tens.

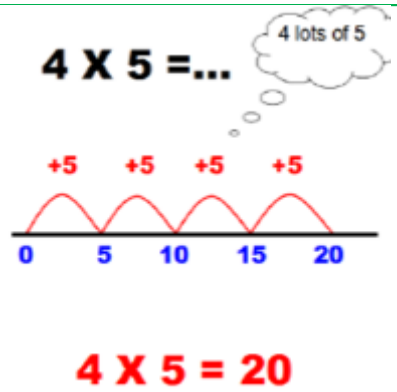
Begin to understand doubling using concrete objects and pictorial representations

Multiplication x

Year 2 Multiply using arrays and repeated addition and write them using the multiplication (x) and equals (=) signs. (using at least 2s, 5s and 10s)

Use repeated addition on a number line:

Starting from zero, make equal jumps up on a number line to work out multiplication facts and write multiplication statements using x and = signs.



Use arrays:

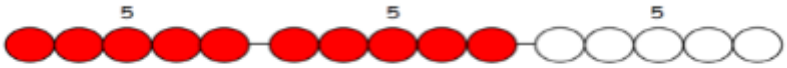
5 x 3 = 15

3 x 5 = 15

5 x 3 = 3 + 3 + 3 + 3 = 15

3 x 5 = 5 + 5 + 5 = 15

5 x 3 = 5 + 5 + 5



Use practical apparatus.

Use mental recall: Children should begin to **recall multiplication facts for 2, 5 and 10** times tables through practice in counting and understanding of the operation.

It is vital that Y2 pupils are taught rapid recall of at least the 2s, 5s and 10s tables facts in order for them to be ready for the statutory tables check at the end of year 4

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times...

Key skills for multiplication at Y2:

Count in steps of 2, 3 and 5 from zero, and in 10s from any number.

Recall and use multiplication facts from the 2, 5 and 10 multiplication tables, including recognising odds and evens.

Write and calculate number statements using the x and = signs.

Show that multiplication can be done in any order (commutative).

Solve a range of problems involving multiplication, using concrete objects, arrays, repeated addition, mental methods, and multiplication facts.

Pupils use a variety of language to discuss and describe multiplication.

http://www.youtube.com/watch?v=YPWmOVt8vgw Teaching for understanding of multiplication facts

Multiplication x

Year 3 **Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, progressing to formal written methods.***

**Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4*

It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.

Initially, children will continue to use arrays where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10),

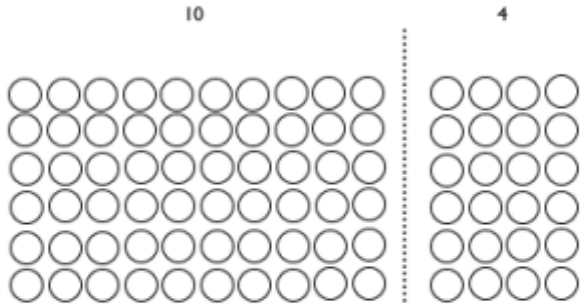
To utilise further methods, children need to:

a) know their multiplication facts up to 10 x 10

b) be able to identify and use related calculations and place value effectively.

<u>Introduce the grid method for multiplying 2-digit by single-digits:</u>

As they progress to multiplying a two-digit number by a single digit number, children should use their knowledge of partitioning two digit numbers into tens and units/ones to help them. For example, when calculating 14 x 6, children should set out the array, then partition the array so that one array has ten columns and the other four.



Partitioning in this way, allows children to identify that the first array shows 10 x 6 and the second array shows 4 x 6. These can then be added to calculate the answer:

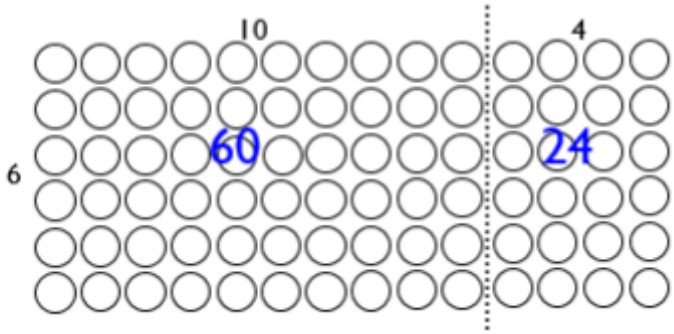
(6 x 10) + (6 x 4)

= 60 + 24

= 84

NB There is no requirement for children to record in this way, but it could be used as a jotting to support development if needed.

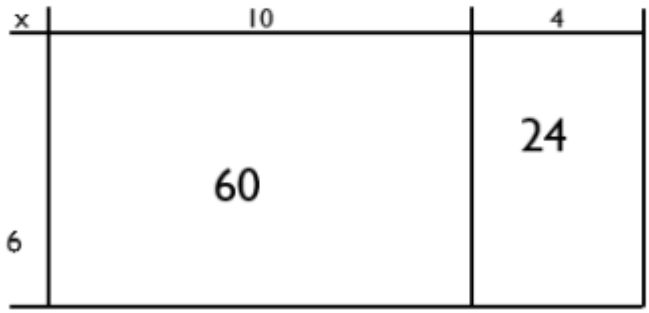
This method is the precursor step to the grid method. Using a two-digit by single digit array, they can partition as above, identifying the number of rows and the number of columns each side of the partition line.



Multiplication x

Year 3 **Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, progressing to formal written methods.***

By placing a box around the array, as in the example below, and by removing the array, the grid method can be seen.



It is really important that children are confident with representing multiplication statements as arrays and understand the rows and columns structure before they develop the written method of recording.

From this, children can use the grid method to calculate two-digit by one-digit multiplication calculations, initially with two digit numbers less than 20. Children should be encouraged to set out their addition in a column at the side to ensure the place value is maintained. When children are working with numbers where they can confidently and correctly calculate the addition mentally, they may do so.

13 x 8

x	10	3
8	80	24

```
   80
+  24
  ---
  104
```

When children are ready, they can then progress to using this method with other two-digit numbers.

37 x 6

x	30	7
6	180	42

```
   180
+   42
  ----
   222
```

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

Multiplication x

Year 3 **Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, progressing to formal written methods.***

To do this, children must be able to:

Partition numbers into tens and units.

Multiply multiples of ten by a single digit (e.g. 20 x 4) using their knowledge of multiplication facts and place value.

Recall and work out multiplication facts in the **2, 3, 4, 5, 8 and 10** times tables.

Work out multiplication facts not known by repeated addition or other taught mental strategies.

IF pupils are ready and are showing a deep understanding when using the grid method, they may be able to be introduced to the formal algorithm in a guided manner

Moving to a formal method

23 x 8 = 184

```
  2 3
X     8
  1 8 4
    2
```

Pupils could be asked to work out a given calculation using the grid, and then compare it to "your" column method. What are the similarities and differences? Unpick the steps and show how it reduces the steps.

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times, _times as big as, once, twice, three times..., **partition, grid method, multiple, product, tens, units, value**

Key skills for multiplication:

Recall and use multiplication facts for the **2, 3, 4, 5, 8 and 10** multiplication tables, and multiply multiples of 10.

It is vital that Y3 pupils are taught rapid recall of at least the 3s, 4s and 8s tables facts and continue to practice and build upon the 2s, 5s and 10s which are covered in Y2 in order for them to be ready for the statutory tables check at the end of year 4

Write and calculate number statements using the multiplication tables they know, including **2-digit x single-digit**, drawing upon mental methods, and progressing to reliable written methods.

Solve multiplication problems, including missing number problems.

Develop mental strategies using commutativity (e.g. 4 x 12 x 5 = 4 x 5 x 12 = 20 x 12 = 240)

Solve simple problems in contexts, deciding which operations and methods to use.

Develop efficient mental methods to solve a range of problems e.g using commutativity (**4 × 12 × 5** = 4 × 5 × 12 **= 20 × 12** = 240) and for missing number problems ? x 5 = 20, 3 x ? = 18, ? x = 32

Year 4 Multiply 2 and 3-digits by a single digit, using all multiplication tables up to 12 x 12

Multiplication x

Developing the grid method

Children will move to Y4 using whichever method they were using as they transitioned from Y3. They will further develop their knowledge of the grid method to multiply any two-digit by any single-digit number, e.g.

79 x 8

x	70	9
8	560	72

```
   560
+   72
  ----
   632
```

Encourage column addition to add accurately.

To support the grid method, children should develop their understanding of place value and facts that are linked to their knowledge of tables. For example, in the calculation above, children should use their knowledge that 7 x 8 = 56 to know that 70 x 8 = 560.

By the end of the year, they will extend their use of the grid method to be able to multiply three-digit numbers by a single digit number, e.g.

346 x 8

x	300	40	6
8	2400	320	48

```
  2400
+  320
+   48
  ----
  2768
```

When children are working with numbers where they can confidently and correctly calculate the addition (or parts of the addition) mentally, they may do so.

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

Move onto **short multiplication** **<u>if</u>** and when children are confident and accurate multiplying 2 and 3-digit numbers by a single digit this way, **and** are already confident in "carrying" for written addition.

```
     3 4 2
X        7
  -------
  2 3 9 4
  -------
        2 1
```

Year 4 Multiply 2 and 3-digits by a single digit, using

all multiplication tables up to **12 x 12**

Multiplication x

Children should be able to:

Approximate before they calculate, and make this a regular part of their calculating, going back to the approximation to check the reasonableness of their answer. e.g:

346 x 9 is approximately 350 x 10 = **3500**

Record an approximation to check the final answer against.

Multiply multiples of ten and one hundred by a single-digit, using

their multiplication table knowledge.

Recall all times tables **up to 12 x 12**

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, groups of, sets of, lots of, equal groups, times, multiply, times as big as, once, twice, three times... partition, grid method, total, multiple, product, sets of, **inverse**

Key skills for multiplication at Y4:

Count in multiples of 6, 7, 9, 11, 12, 25 and 1000

Recall multiplication facts for **all multiplication tables up to 12 x 12.**

Use place value, known facts and derived facts to multiply mentally, e.g.

multiply by 1, 10, 100, by 0, or to multiply 3 numbers.

Use commutativity and other strategies mentally 3 x 6 = 6 x 3 , 2 x 6 x 5 = 10 x 6 , 39x7 = 30 x 7 + 9 x 7.

Solve problems with increasingly complex multiplication in a range of contexts.

Recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)

Year 5 Multiply up to 4-digits by 1 digit moving onto by 2 digits as and where appropriate

Multiplication x

Continuing with the assumption that pupils will use one digit one square, will use place holders as appropriate to 'even up' calculations and that the decimal point will straddle the line rather than have its own square *(as it has no inherent place value)*

Short multiplication

Introduce by comparing a grid method calculation to a short multiplication method, to see how the steps are related, but notice how there are fewer steps involved in the column method.

Children need to be taught to approximate first, e.g. for 72 x 38, they will use rounding: 72 x 38 is approximately 70 x 40 = 2800, and use the approximation to check the reasonableness of their answer against.

Short multiplication for multiplying by a single digit

Children should be taught to use short multiplication from the start of year 5 as shown below:

24 × 6 becomes

```
      2 4
  ×     6
  -------
  1 4 4
    2
```

342 × 7 becomes

```
      3 4 2
  ×       7
  ---------
  2 3 9 4
    2 1
```

2741 × 6 becomes

```
      2 7 4 1
  ×         6
  -----------
  1 6 4 4 6
    4 2
```

When multiplying decimal numbers by a single digit they should understand that 6.0 is of equal value to 6 eg:

23.4 x 6 becomes

```
   2 3 . 4
 x   6 . 0
 -----------
 1 4 0 . 4
   2 2
```

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

If pupils show sufficient understanding when using short multiplication to solve problems of increasing complexity, they may move towards long multiplication as set out in the Y6 section.

Year 5 **Multiply up to 4-digits by 1 digit moving onto by 2 digits as and where appropriate**

Multiplication x

Key vocabulary groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, _times as big as, once, twice, three times..., partition, grid method, total, multiple, product, inverse, **square, factor, integer, decimal, short/long multiplication, carry**

Key skills for multiplication at Y5:

Identify multiples and factors, using knowledge of **multiplication tables to 12x12.**

Multiply and divide integers and decimals by 10, 100 and 1000.

Recognise and use square and cube numbers and their notation.

Solve problems involving combinations of operations, choosing and using calculations and methods appropriately.

Year 6 Short and long multiplication including multiplication of decimals with up to 2d.p by a single digit.

Multiplication x

Continuing with the assumption that pupils will use one digit one square, will use place holders as appropriate to 'even up' calculations and that the decimal point will straddle the line rather than have its own square *(as it has no inherent place value)*

By the end of Y6, children should be able to use the long multiplication to multiply any number by a two-digit number.

Please note: For long multiplication, the children should start with the **least significant digits first (i.e. start with the units/ones). When using this method, we carry the numbers **above** the calculation.*

24 x 16 becomes

```
     2
    24
x   16
   144
+  240
   384
```

124 x 26 becomes

```
    1 2
   124
x   26
   744
+ 2480
  3224
   1 1
```

They should also develop the method to be able to multiply decimal numbers with up to two decimal places, e.g.

124 x 2.6 becomes

```
     1 2
   124.0
x    2.6
    74.4
+  248.0
   322.4
     1 1
```

Children should also be using this method to solve problems and multiply numbers, including those with decimals, in the context of money or measures, e.g. to calculate the cost of 7 items at £8.63 each, or the total length of six pieces of ribbon of 2.28m each.

Year 6 Short and long multiplication including multiplication of decimals with up to 2d.p by a single digit.

Multiplication x

Children will be able to:

Use rounding and place value to make approximations before calculating and use these to check answers against.

Use **short multiplication** to multiply numbers with **more than 4-digits by a single digit**; to multiply money and measures, and to **multiply decimals with up to 2d.p. by a single digit.**

Use **long multiplication** to multiply numbers with **at least 4 digits by a 2-digit number.**

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times… partition, grid method, total, multiple, product, inverse, square, factor, integer, decimal, short / long multiplication, carry, **tenths, hundredths, decimal**

Key skills for multiplication at Y6:

Recall multiplication facts for all times tables up to **12 x 12 (as Y4 and Y5).**

Multiply multi-digit numbers, up to 4-digit x 2-digit using long multiplication.

Perform mental calculations with mixed operations and large numbers.

Solve multi-step problems in a range of contexts, choosing appropriate combinations of operations and methods.

Estimate answers using round and approximation and determine levels of accuracy.

Round any integer to a required degree of accuracy.

Division ÷

Division

In developing a written method for division, it is important that children understand the concept of division, in that it is:

repeated subtraction
sharing into equal amounts
grouping

They also need to understand and work with certain principles, i.e. that it is:

the inverse of multiplication
not commutative i.e. 15 ÷3 is not the same as 3 ÷ 15
not associative i.e. 30 ÷ (5 ÷ 2) is not the same as (30 ÷ 5) ÷ 2

It is important to use correct vocabulary when talking about division with the children - quotient (answer), divisor (what you are dividing by) and dividend (what you are dividing)

YR

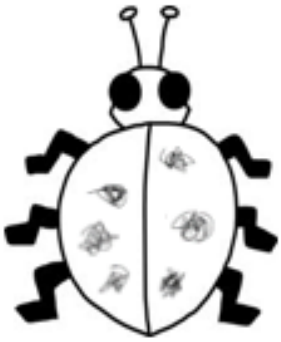
Early Learning Goal:

Children solve problems, including halving and sharing.

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, including small world play, role play, counters, cubes etc.

Children may also investigate sharing items or putting items into groups using items such as egg boxes, ice cube trays and baking tins which are arrays.

They may develop ways of recording calculations using pictures, etc.



A child's jotting showing halving six spots between two sides of a ladybird.

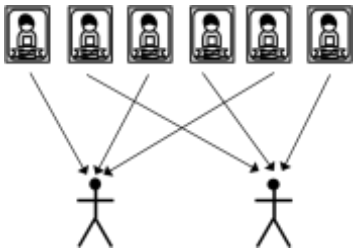
A child's jotting showing how they shared the apples at snack time between two groups.

Year 1 Group and share small quantities

Division

Using objects, diagrams and pictorial representations to solve problems involving **both grouping and sharing**.

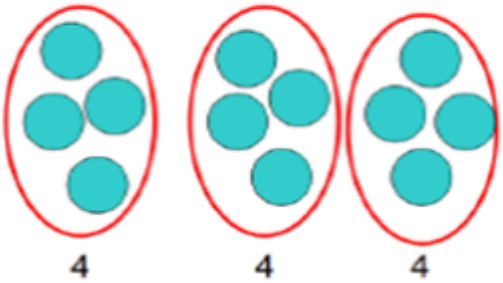
In year one, children will continue to solve division problems using practical equipment and jottings. They should use the equipment to share objects and separate them into groups, answering questions such as 'If we share these six apples between the three of you, how many will you each have? How do you know?' or 'If six football stickers are shared between two people, how many do they each get?



They solve sharing problems by using a 'one for you, one for me' strategy until all of the items have been given out.

How many groups of 4 can be made with 12 stars? = **3**

Grouping



Sharing

12 shared between 3 is 4

Year 1 Group and share small quantities

Division

Once children have mastered this approach they should be encouraged to begin to calculate mathematical statements for division within the multiplication tables and write them using the division (÷) and equals (=) signs.

For this, children will utilise practical equipment to represent division calculations as grouping (repeated subtraction) and use jottings to support their calculation, e.g.

12 ÷ 3 =

Children need to understand that this calculation reads as 'How many groups of 3 are there in 12?'

If ready, they may also continue to develop their knowledge of division with remainders, e.g.

13 ÷ 4 =

13 ÷ 4 = 3 remainder 1

Children should be encouraged to make decisions about what to do with remainders after division and round up or down accordingly. In the calculation 13 ÷ 4, the answer is 3 remainder 1, but whether the answer should be rounded up to 4 or rounded down to 3 depends on the context, as in the examples below:

I have £13. Books are £4 each. How many can I buy?

Answer: 3 (the remaining £1 is not enough to buy another book)

Apples are packed into boxes of 4. There are 13 apples. How many boxes are needed?

Answer: 4 (the remaining 1 apple still needs to be placed into a box)

Year 1 Group and share small quantities

Division

Pupils should :

use lots of practical apparatus, arrays and picture representations

Be taught to understand the difference between "grouping" objects (How many groups of 2 can you make?) and "sharing" (Share these sweets between 2 people)

Be able to count in multiples of 2s, 5s and 10s.

Find **half** of a group of objects by sharing into 2 equal groups.

Key Vocabulary: share, share equally, one each, two each…, group, groups of, lots of, array

Key number skills needed for division at Y1:

Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations arrays with the support of the teacher.

Through grouping and sharing small quantities, pupils begin to understand, division, and finding simple fractions of objects, numbers and quantities.

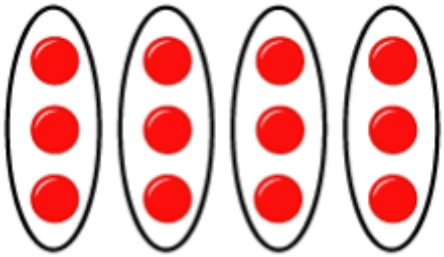
They make connections between arrays, number patterns, and counting in twos, fives and tens.

Year 2 Group and share, using the ÷ and = sign

Division

Use objects, arrays, diagrams and pictorial representations, and grouping on a number line.

Arrays



12 ÷ 3 = 4

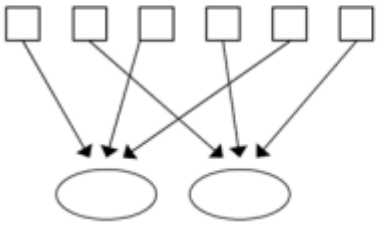
This represents **12 ÷ 3, posed as** how many groups of 3 are in 12? Pupils should also show that the same array can represent **12 ÷ 4 = 3** if grouped horizontally.

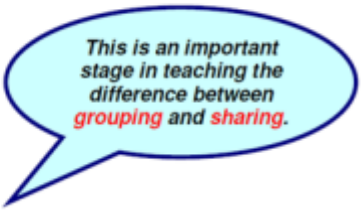
Know and understand sharing and grouping:

Grouping

There are 6 sweets, how many people can have 2 sweets each?

6 sweets shared between 2 people, how many do they each get?





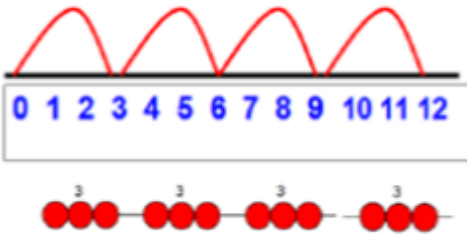
Sharing

Pose **12 ÷ 3** as "**How many groups of 3 are in 12?**"

Grouping using a number line:

Group from zero in equal jumps of the divisor to find out how many groups of _ in _ ?.

Pupils could use a bead string or practical apparatus to work out problems like "A CD costs £3. How many CDs can I buy with £12" **This is an important method to develop understanding of division as grouping.**



12 ÷ 3 = 4

Year 2 Group and share, using the ÷ and = sign

DIVISION

Once this understanding is secure, move onto using more complex numbers which may require practical resources coupled with a numberline.

In order to develop the children's understanding of division, children should first use the repeated subtraction on a vertical number line alongside the continued use of practical equipment. There are two stages to this:

Stage 1 – repeatedly subtracting individual groups of the divisor

Stage 2 – subtracting multiples of the divisor (initially 10 groups and individual groups, then 10 groups and other multiples in line with tables knowledge)

After each group has been subtracted, children should consider how many are left to enable them to identify the amount remaining on the number line.

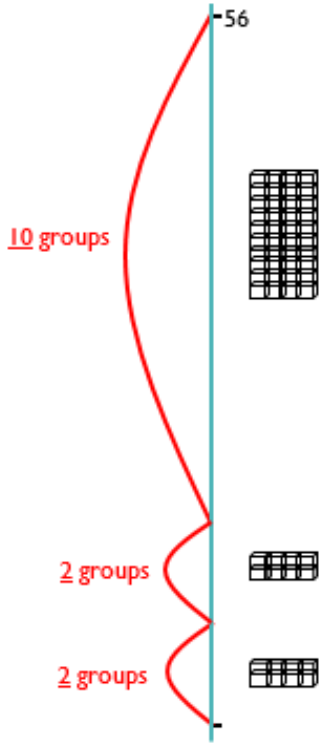
Stage 1

56 ÷ 4 = 14 (groups of 4)

Stage 2

56 ÷ 4 = 10(groups of 4) + 2(groups of 4) + 2(groups of 4

= 14(groups of 4)



Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

Year 2 Group and share, using the ÷ and = sign

Division

Key Vocabulary: **share, share equally, one each, two each…, group, equal groups of, lots of,**

array, divide, divided by, divided into, division, grouping, number line, left, left over

Key number skills needed for division at Y2:

☐ Count in steps of 2, 3, and 5 from 0

☐ Recall and use multiplication and division facts for the **2, 5 and 10** multiplication tables, including recognising odd and even numbers.

☐ Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the x, ÷ and = signs.

☐ Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.

☐ Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Year 3 Divide 2-digit numbers by a single digit using the 'short' written method of division

Division

Children will start Y3 using whichever method they were used to in Y2

Use this as a starting point and to establish levels of understanding but move pupils onto the Y3 method as soon as possible

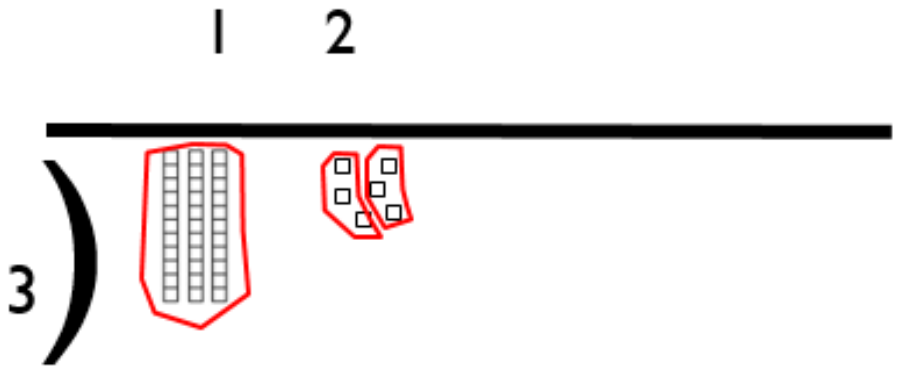
Children will move onto using the 'short' method of division (known as 'bus-stop' method)

This method should initially be introduced using practical apparatus such as 'Base 10' to represent the dividend (what you are dividing) and sharing the rods and cubes into groups of the divisor (what you are dividing it by)

36 ÷ 3

"The divisor is 3 so let's sort the 'tens rods' into groups of 3 – how many can we make?" (1 group)

"Now let's sort the 'ones cubes' into groups of 3 – how many can we make?" (2 groups)



See next page for development in this method

Year 3 Divide 2-digit numbers by a single digit using the 'short' written method of division

Division

This approach can then be used to develop an understanding that 'carrying' is in fact an exchange.

E.g. Once the rods have been grouped, if there is a leftover rod or if a group couldn't be made, then you exchange the rods into cubes in the next column and group them, if there are leftover cubes after this, then they are the remainder - this makes the bus stop method make sense with a highly practical, hands on approach. For example:

46 ÷ 3

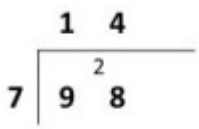
Step 1

Step 2 1

Step 3 1 5 r 1

Once this practical approach is secure, the children should be moved onto the more formal written method – you should ensure that the children understand the place value of each digit when working through these calculations.

98 ÷ 7 becomes



Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

Year 3 Divide 2-digit numbers by a single digit using the 'short' written method of division

Division

Key Vocabulary: divisor, dividend, quotient, share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, remainder, multiple

Key number skills needed for division at Y3:

☐ Recall and use multiplication and division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables (through doubling,

connect the 2, 4 and 8s).

☐ Write and calculate mathematical statements for multiplication and division using the multiplication tables

that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal

written methods.

☐ Solve problems, in contexts, and including missing number problems, involving multiplication and division.

☐ Pupils develop efficient mental methods, for example, using multiplication and division facts (e.g. using 3 × 2 =

6, 6 ÷ 3 = 2 and 2 = 6 ÷ 3) to derive related facts (30 × 2 = 60, so 60 ÷ 3 = 20 and 20 = 60 ÷ 3).

☐ Pupils develop reliable written methods for division, starting with calculations of 2-digit numbers by 1-digit numbers and progressing to the formal written method of short division.

Year 4 ***Divide numbers up to 3 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.***

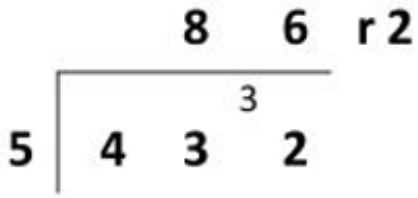
Division

Continue to develop short division

For calculations where numbers with up to 4 digits are divided by a single digit number, children are expected to use short division.

Children should continue to develop their fluency with short division and develop their understanding by encountering remainders more frequently and working with dividends of up to 3 digits

432 ÷ 5 becomes



Children should continue to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

Key Vocabulary: divisor, dividend, quotient, share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, carry, remainder, multiple, divisible by, factor

Key number skills needed for division at Y4:

Recall multiplication and division facts for all numbers up to 12 x 12.

Use place value, known and derived facts to multiply and divide mentally, including: multiplying and dividing by 10 and 100 and 1.

Pupils practise to become fluent in the formal written method of short division with exact answers when dividing by a one-digit number.

Pupils practise mental methods and extend this to three-digit numbers to derive facts, for example 200 × 3 = 600 so 600 ÷ 3 = 200

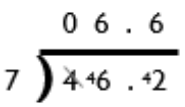
Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children.

Division

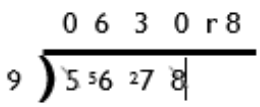
Year 5 Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context. Begin to develop the use of long division to divide 2 or 3 digit numbers by 2 digit numbers

Children should continue to develop their use of short division by working with dividends of up to 4 digits and decimal dividends.

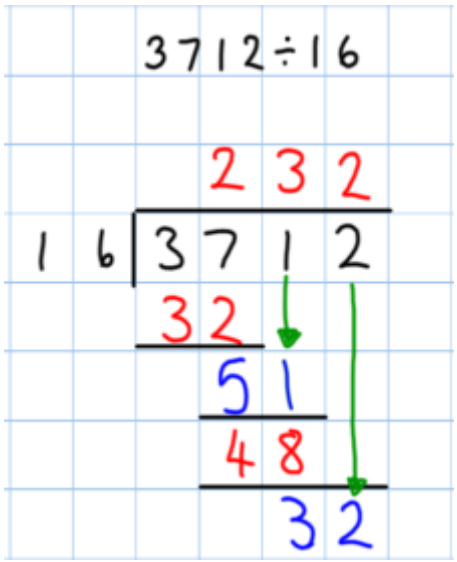
46.2 ÷ 7 becomes



5678 ÷ 9 becomes



In addition to their continued development of the short method of division, children should learn to use long division when the divisor is over 12



Number facts to help:

16 (1x16)

32 (2x 16)

48 (3x 16)

64 (4x16)

80 (5x16)

96 (6x16)

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

Long division works from left to right.

How many times will 16 go into 3? This can't be done, so we say:

How many times will 16 go into 37?

Answer = 2 This is written above the 7.

2x16 =32 so we subtract 32 from 37, this leaves us with 5.

Bring down the 1 to give us a new target of 51.

How many 16's in 51?

Answer = 3 This is written above the 1.

3 x 16 = 48 (we cannot go past our target of 51) so we subtract 48 from 51, this leaves us with 3.

Bring down the 2 to give us a new target of 32.

How nay 16's in 32? Answer = 2 this is written above the 2.

Since 2 x16 = 32, there are no remainders and no further numbers to bring down.

Year 5 Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context. Begin to develop the use of long division to divide 2 or 3 digit numbers by 2 digit numbers

Division

Key Vocabulary: divisor, dividend, quotient, share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, "carry", remainder, multiple, divisible by, factor, inverse, prime number, prime factors,

composite number (non-prime)

Key number skills needed for division at Y5:

Recall multiplication and division facts for all numbers up to 12 x 12 (as in Y4).

Multiply and divide numbers mentally, drawing upon known facts.

Identify multiples and factors, including finding all factor pairs of a number, and common factors of two number.

Solve problems involving multiplication and division where larger numbers are decomposed into their factors.

Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.

Use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.

Work out whether a number up to 100 is prime, and recall prime numbers to 19.

Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

Use multiplication and division as inverses.

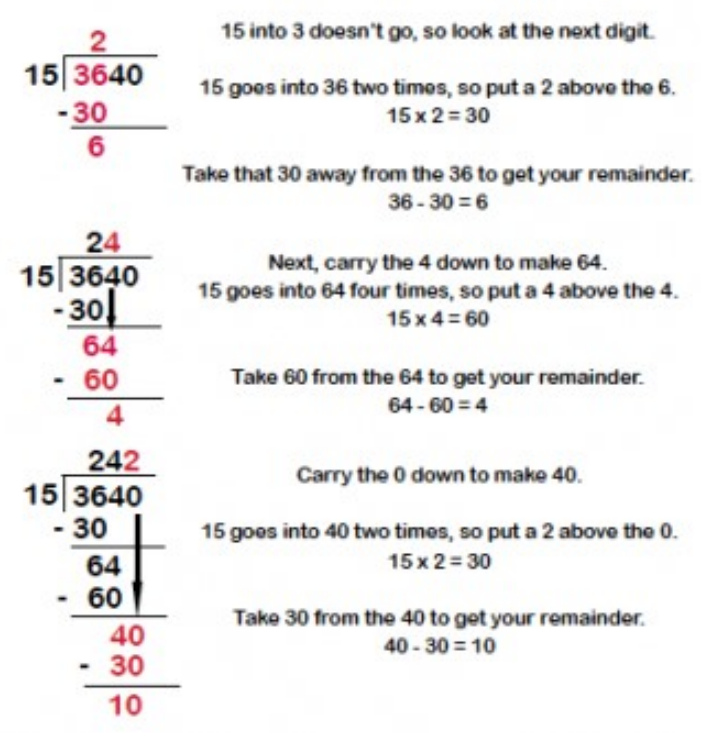
Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (e.g. 98 ÷ 4 = 24 r 2 = 241/2 = 24.5 ≈ 25).

Solve problems involving combinations of all four operations, including understanding of the equals sign, and including division for scaling by different fractions and problems involving simple rates.

Year 6 **Divide numbers up to 4 digits by a two-digit number using the correct formal written method of division where appropriate, interpreting remainders according to the context.**

Division

Children should continue to develop their use of both methods of division using increasingly complex numbers – they should begin to show remainders as fractions and, if ready, move onto showing remainders as decimals up to 2dp



In this example, 10 is the remainder so the answer could be written as **242 r 10**.

To show the remainder as a fraction, you make the remainder the numerator and the divisor the denominator so in this case the answer would be **242 $^{10}/_{15}$** which simplifies to **242 $^{2}/_{3}$**

We could then use our knowledge of decimal equivalents ($^{2}/_{3}$ = 0.66) to give the answer as a decimal answer of **242.66**

Pupils only need to give remainders as fractions or decimals when the questions specifically requests it. In all other cases, each of these answers would be perfectly acceptable and this approach is intended to extend those who are ready for it.

To show the remainder as a decimal relies upon children's knowledge of decimal fraction equivalents. For decimals with no more than 2 decimal places, they should be able to identify:

Half: $\frac{1}{2}$ = 0.5

Quarters: $\frac{1}{4}$ = 0.25, $\frac{3}{4}$ = 0.75

Thirds: $^{1}/_{3}$ = 0.33' $^{2}/_{3}$ = 0.66'

Fifths: $\frac{1}{5}$ = 0.2, $\frac{2}{5}$ = 0.4, $\frac{3}{5}$ = 0.6, $\frac{4}{5}$ = 0.8

Tenths: $\frac{1}{10}$ = 0.1, $\frac{2}{10}$ = 0.2, $\frac{3}{10}$ = 0.3, $\frac{4}{10}$ = 0.4, $\frac{5}{10}$ = 0.5, $\frac{6}{10}$ = 0.6, $\frac{7}{10}$ = 0.7, $\frac{8}{10}$ = 0.8, $\frac{9}{10}$ = 0.9

and reduce other equivalent fractions to their lowest terms.

Year 6 **Divide numbers up to 4 digits by a two-digit number using the correct formal written method of division where appropriate, interpreting remainders according to the context.**

Division

Key Vocabulary: As previously, & common factor

Key number skills needed for division at Y6:

Recall and use multiplication and division facts for all numbers to 12 x 12 for more complex calculations

Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for

the context. Use short division where appropriate.

Perform mental calculations, including with mixed operations and large numbers.

Identify common factors, common multiples and prime numbers.

Solve problems involving all 4 operations.

Use estimation to check answers to calculations and determine

accuracy, in the context of a problem.

Use written division methods in cases where the answer has up to two decimal places.

Solve problems which require answers to be rounded to specified degrees of accuracy.

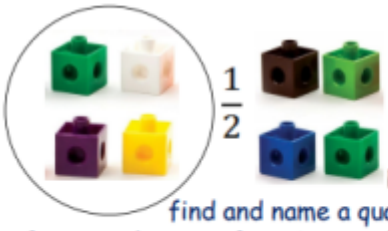
Fractions

Year 1 Fractions

How can we progress with fractions?

Recognise, find and name a half as one of two equal parts of an object, shape or quantity.

Concrete



$\frac{1}{2}$

Pictorial

A whole apple

1

Half an apple

$\frac{1}{2}$

Abstract

Half of 10 = ☐

Half of 8 = ☐

of 14 = ☐

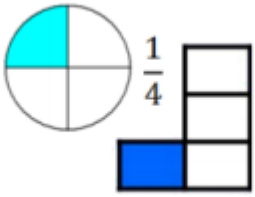
$\frac{1}{2}$ ☐ of

Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.

Concrete

$\frac{1}{4}$

Pictorial



$\frac{1}{4}$

Abstract

A quarter of 20 = ☐

A quarter of 12 = ☐

of 8 = ☐

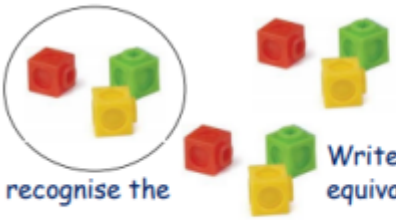
$\frac{1}{4}$ ☐

Year 2 Fractions

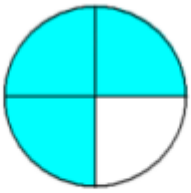
How can we progress with fractions?

Recognise, find and name and write fractions 1/3, 1/4, 2/4 and 3/4 of a length, shape, set of objects or quantity.

Concrete

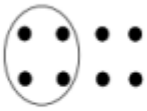


Pictorial



Abstract

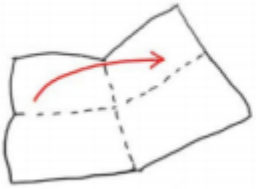
$\frac{2}{4}$ of 8 = ☐



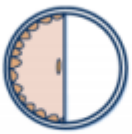
and recognise the 1/2.

Write simple equivalence fractions of 2/4 and

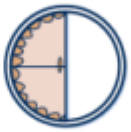
Concrete



Pictorial



I have $\frac{1}{2}$ a pie



You have $\frac{2}{4}$ of a pie

Abstract

$\frac{1}{2}$ of 6 = ☐

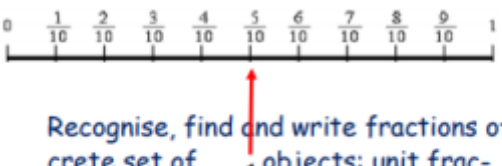
Fractions

Year 3 Fractions

How can we progress with fractions?

Count up and down in tenths: recognise that tenths arise from dividing an object into ten equal parts and in dividing one-digit numbers or quantities by ten.

Concrete



Pictorial

$\frac{3}{10}$ $\frac{3}{10}$ $\frac{3}{10}$

Abstract

$\frac{1}{10}$ of 6 = 0.6 because 6 ÷ 10 = 0.6

$\frac{1}{10}$ of 7 = 0.7 because 7 ÷ 10 = 0.7

Recognise, find and write fractions of a discrete set of objects: unit fractions and use and non-unit fractions as numbers.

Concrete

$\frac{1}{8}$

Pictorial

$\frac{3}{4}$

Abstract

$\frac{1}{5}$ of 15 sweets = 3 because 15 ÷ 5 = 3

$\frac{2}{5}$ of 15 sweets = 6 because 15 ÷ 5 = 3 and 3 x 2 = 6

Recognise and show, using diagrams, equivalent fractions with small denominators.

Concrete

two halves $\frac{2}{2}$ four quarters $\frac{4}{4}$

Pictorial

$\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$ $\frac{4}{8}$

Abstract

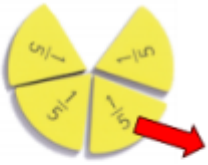
Sam says that two quarters is the same as one half.

Is he correct?

How do you know?

Add and subtract fractions with the same denominator.

Concrete



$\frac{1}{5}$

Pictorial

$\frac{1}{5}$ + $\frac{3}{5}$ = $\frac{4}{5}$

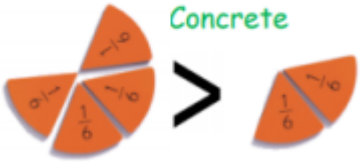
Abstract

$\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$

$\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$

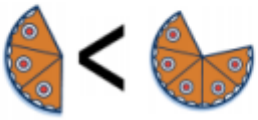
Compare and order unit fractions with the same denominators.

Concrete



$\frac{1}{6}$ > $\frac{1}{6}$

Pictorial



Abstract

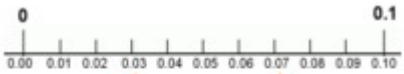
$\frac{2}{8}$ $\frac{3}{8}$ $\frac{5}{8}$ $\frac{7}{8}$

Year 4 Fractions

How can we progress with fractions?

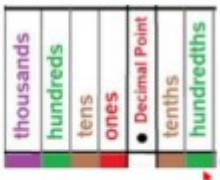
Count up and down in hundredths: recognise that hundredths arise when dividing an object by 100 and dividing tenths by 10.

Concrete



Recognise and write decimal equivalents to $\frac{3}{100}$ 1/2, 1/4 $\frac{7}{100}$ and 3/4.

Pictorial



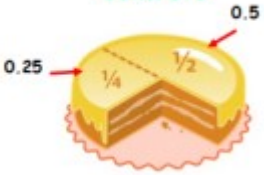
1 hundredth = 0.01 = $\frac{1}{100}$

Abstract

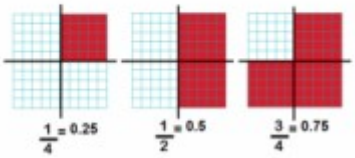
$\frac{1}{100}$ of 60 = 0.6
because 60 ÷ 100 = 0.6

$\frac{1}{10}$ of 70 = 0.7
so $\frac{1}{100}$ of 70 = 0.07

Concrete



Pictorial



Abstract

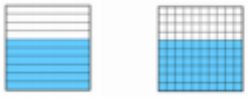
$\frac{1}{2}$ = 0.5
$\frac{1}{4}$ = 0.25
$\frac{3}{4}$ = 0.75

Recognise and write decimal equivalents of any number of tenths or hundredths.

Concrete

Recognise $\frac{1}{10}$ of the chocolate bar = 0.1

Pictorial



0.6 six tenths

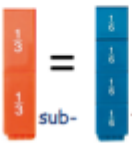
0.60 sixty hundredths

Abstract

$\frac{1}{10}$ = 0.1
$\frac{3}{10}$ = 0.3
$\frac{5}{10}$ = $\frac{1}{2}$ = 0.5
$\frac{8}{100}$ = 0.08

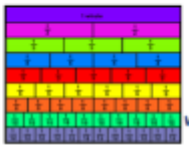
and show, using diagrams, families of common equivalents.

Concrete



Add and subtract fractions with the same denominator.

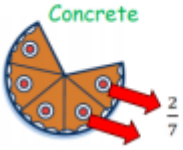
Pictorial



Abstract

$\frac{2}{3}$ = $\frac{4}{6}$
$\frac{3}{5}$ = $\frac{6}{10}$
$\frac{2}{12}$ = $\frac{1}{6}$

Concrete



$\frac{2}{7}$

Pictorial

$\frac{1}{5}$ + $\frac{3}{5}$ = $\frac{4}{5}$

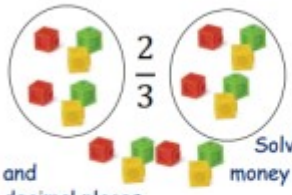
Abstract

Sam eats $\frac{2}{7}$ of a whole pizza. How much pizza does he have left?

Lucy and Ben both eat $\frac{3}{8}$ of a cake. How much have they eat altogether?

Solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number.

Concrete



$\frac{2}{3}$

Pictorial

$\frac{2}{3}$ $\frac{1}{3}$

Abstract

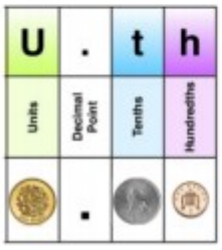
$\frac{2}{3}$ of £18

£18 ÷ 3 = £6
£6 x 2 = £12

Solve simple measure and money problems involving fractions and decimals to two decimal places.

Concrete

Pictorial



Abstract

100cm = 1m

50cm = $\frac{1}{2}$ = 0.5m

25cm = $\frac{1}{4}$ = 0.25m

10cm = $\frac{1}{10}$ = 0.1m

30cm = $\frac{3}{10}$ = 0.3m

Fractions

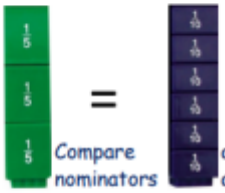
Fractions

Year 5 Fractions

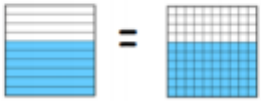
How can we progress with fractions?

Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths.

Concrete



Pictorial



$\frac{6}{10} = \frac{60}{100}$

Abstract

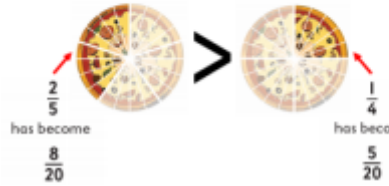
$\frac{3}{5} = \frac{6}{10} = \frac{60}{100}$

$\frac{3}{4} = \frac{75}{100}$

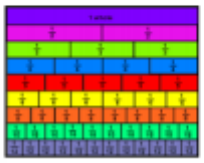
$\frac{1}{5} = \frac{2}{10} = \frac{20}{100}$

Compare and order fractions whose denominators are all multiples of the same number.

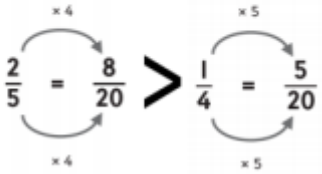
Concrete



Pictorial

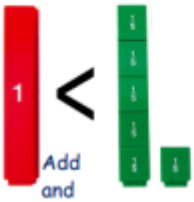


Abstract

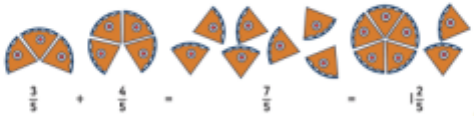


Recognise mixed numbers and improper fractions. Convert from one form to the other and write mathematical statements >1 as a mixed number.

Concrete



Pictorial



Abstract

$\frac{7}{2} = 3\frac{1}{2}$

because 7 ÷ 2 = 3 with 1 half left over

$2\frac{1}{3} = \frac{7}{3}$

because 2 x 3 = 6 with 1 third left to add

Add and subtract fractions with the same denominators and denominators that are multiples of the same numbers.

Concrete

Pictorial

Abstract

$\frac{2}{5} - \frac{1}{4}$

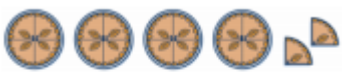
So, $\frac{8}{20} + \frac{5}{20} = \frac{13}{20}$

So, $\frac{8}{20} - \frac{5}{20} = \frac{3}{20}$

Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams.

Concrete

Pictorial



Abstract

Multiply a proper fraction by a whole number:

$\frac{3}{4} \times 6 = \frac{18}{4}$

Change to a mixed number:

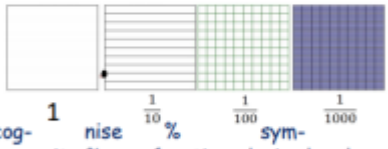
$\frac{18}{4} = 4\frac{2}{4}$

6 lots of $\frac{3}{4}$ to $4\frac{2}{4}$ altogether

Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents.

Concrete

Pictorial



Abstract

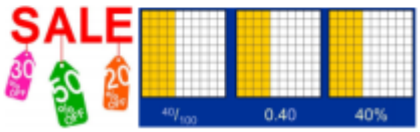
67.153

How many thousandths does this number have? How many more thousandths do you need to add to make 67.16?

Recognise % symbol and understand the meaning: write % as a fraction, decimal and percentage.

Concrete

Pictorial



Abstract

$\frac{4}{10} = 40\% = 0.4$

$\frac{32}{100} = 32\% = 0.32$

$\frac{75}{100} = 75\% = 0.75$

$\frac{2}{25} = \frac{8}{100} = 8\% = 0.08$

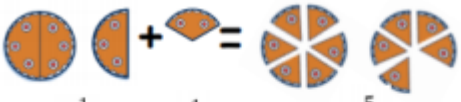
Fractions

Year 6 Fractions

How can we progress with fractions?

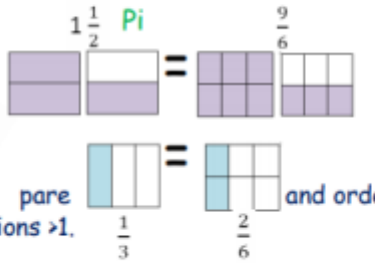
Add and subtract fractions with different denominators and mixed numbers using the concept of equivalent fractions.

Concrete



$1\frac{1}{2}$ Com- $\frac{1}{3}$ including $1\frac{5}{6}$

Pi



pare fractions >1. and order

Abstract

$1\frac{1}{2} + \frac{1}{3} = 1\frac{5}{6}$

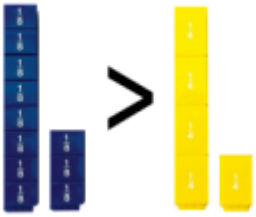
because $1\frac{1}{2} = \frac{3}{2}$

$\frac{3}{2} = \frac{9}{6}$ and $\frac{1}{3} = \frac{2}{6}$

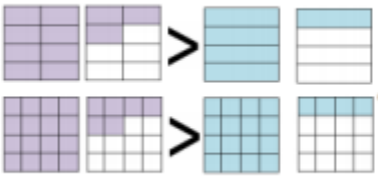
so $\frac{9}{6} + \frac{2}{6} = \frac{11}{6} = 1\frac{5}{6}$

frac-

Concrete



Pictorial



Abstract

Which is greater?

$\frac{2}{8} < \frac{6}{16}$

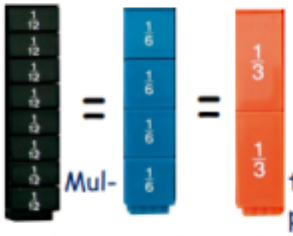
Ordering from smallest to largest by using equivalent fractions:

$\frac{5}{12}, \frac{2}{3}, \frac{5}{6}$

$\frac{5}{12}, \frac{8}{12}, \frac{10}{12}$

Use common factors to simplify fractions; use common multiples to express fractions in the same denomination.

Concrete

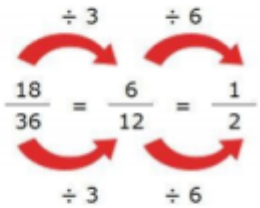


Mul- tiply simple proper frac- ing the answer in its simplest form.

Pictorial

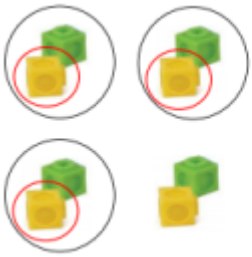
pairs of tions, writ-

Abstract



Concrete

$\frac{1}{2}$ of $\frac{3}{4}$

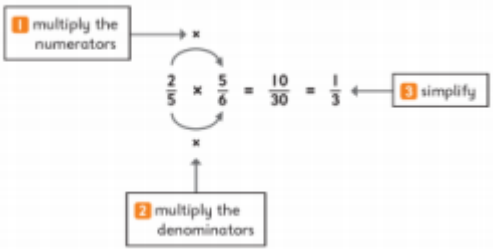


Pictorial

$\frac{1}{2}$ of $\frac{3}{4}$

Abstract

$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$



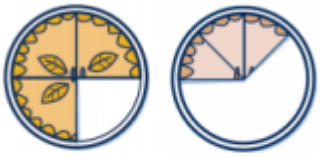
Fractions

Recall and use equivalences between simple fractions, decimals and percentages including in different contexts.

Concrete

Pictorial

Which would you prefer 75% or $\frac{3}{8}$ of a pie?



75% $\frac{3}{8}$

Divide proper fractions by whole numbers.

Abstract

John scored $\frac{40}{80}$ in his spelling test and Hannah scored 40%. Who scored more?

John = $\frac{40}{80}$ = 50%

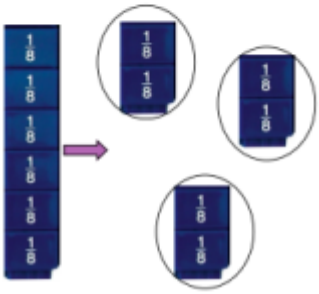
Hannah = 40%

One paving slab is 0.3m long and another is $\frac{1}{4}$ of a metre. Which is longer?

$\frac{1}{4}$ = 0.25m

0.3m is larger than 0.25m

Concrete



Pictorial

$\frac{1}{2} \div 3 = \frac{1}{6}$

Abstract

$\frac{1}{2} \div 3 = \frac{1}{6}$

Keep it, change it, flip it!

$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

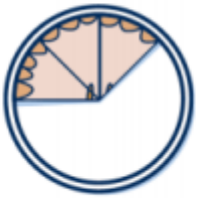
Associate fractions with division and calculate decimal fraction equivalents.

Concrete

quarters

Pictorial

3 slices of pie 'out of' 8



$\frac{3}{8}$

Abstract

$\frac{3}{8}$

3 'out of' 8 is the same as 3 'divided by' 8

3 ÷ 8 = 0·375

So $\frac{3}{8}$ = 0·375